General formula for Multiplication

Multiplication in three simple steps

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Abstract
At first we take a look at the Vedic Mathematics method of the square of an integer and then devise a
two step method to multiply any two integers with the same numeral in the units’ place.

Our main result is the simplest and fastest 3 step method to multiply any two integers without using a
calculator and to derive the general formula for Multiplication, i.e. ‘x’ multiplied by ‘y’.
Let x and y be any two integers that belongs to Z with integers \( u_1 \) and \( u_2 \) in their units place respectively.

Then

\[
x \times y = [(x - u_1)(y - u_2)] + [(x - u_1)u_2] + [(y - u_2)u_1] + u_1u_2.
\]

Multiplying any two integers in three simple steps:

Consider any two integers x and y with \( u_1 \) and \( u_2 \) being the numerals in their unit’s place respectively.

Example 1: 127 * 63

Step 1: Multiply the numerals in the units’ place. Now write the numeral in the unit’s place of the result in
the unit’s place of the answer. Carry the remainder to the next step.

7*3 is 21.

Create an answer column. Answer: 1

Carry 2 to the next step.

Step 2: Now ignore the numeral in the unit’s place and consider the new integers so formed which is 12
and 6. Now multiply these two integers from the numeral in the units’ place of the other integer and add
then together. Add the remainder from the previous step to this result. Now write the numeral in the unit’s
place of this result in the ten’s place of the answer. Carry the remainder to the next step.

i.e. \((12*3) + (6*7) = 36+42 = 78. 78+2 = 80\) so write 0 in the tens’ place of the answer.

Answer: 01.

Carry 8 to the next step

Step 3: multiply the two newly formed digits i.e. 12*6 = 72. Now add the remainder from the previous
step.

72+8 = 80

Answer: 8001.

Example 2: 348 *85
Step 1: $8*5 = 40$. Write 0 in the unit’s place of the answer. \textit{Answer: 0}

Carry 4 to step 2

Step 2: consider 34 and 8. $(34*5) + (8*8) = 170 + 64 = 234$. Now add the remainder from the previous step. $234 + 4 = 238$

Now write 8 in the ten’s place of the answer.

\textit{Answer: 80}

Carry 23 to step 3.

Step 3: $34*8 = 272$. $272 + 23 = 295$.

\textit{Answer: 29580}.

\textbf{Brief Description before deriving the General formula for Multiplication}

I. Let us first look at the basic Vedic Mathematics sutra for Multiplication “Ekaadhikyena Purvena”

Squaring numbers ending in 5:

$25^2 = 2 \times 3 / 25 = 625$

Let us examine closely.

$25^2 = 625$

$2*3 = 6$ and 6 corresponds to 600

This means $600 = 20*30$

$625 = 20*30 + 5^2$

$25^2 = [(25-5) \times (25+5)] + 5^2$

$x^2 = [(x-u)\times(x+u)] + u^2$ This is the general formula for the square of any integer.

Example 1: $124^2 = (124-4)(124+4) + 4^2 = (120*128) + 16 = 15360 + 16 = 15376$.

Example 2: $89^2 = (89-9)(89+9) + 9^2 = (80*98) + 81 = 7840 + 81 = 7921$.

Now we shall extend this SUTRA “Ekaadhikyena Purvena” to multiply any two numbers with the same numeral in the units place.

II. The general formula for multiplying any two integers that end with the same numeral in the units’ place.

Let $x$ and $y$ be two integers that belong to $Z$ ending with 5 in the units place.
Example: 1

75 * 95

Now ignore the numeral in the units’ place which is 5 and consider the new integers so formed.
Consider the example 75 * 95 where x = 75 and y = 95 and 5 = ‘u’ the numeral in the units place.

7*9=63 and the mean of 7 and 9 is 8

63 + 8 = 71 and the answer is 7125.

In this case 71 corresponds to 7100 That is 63 corresponds to 6300 and 8 corresponds to 800

7125 = 6300+800+25

Now 6300 = 70*90
800 = (70+90)*5 = 160*5
25 = 5^2

7125 = (70*90) + [(70+90)*5] + 5^2

75*95 = [(75-5)(95-5)] + ((75-5)+(95-5))*5 + 5^2

x*y = [(x-u)(y-u)] + {[(x-u)+(y-u)]*u} + u^2.

This is the required general formula for multiplying any two integers with the same numeral in the units’ place.

Example: 69*219

9^2 is 81 write 1 in units place. Carry 8 to step 2 Answer: 1

Ignore 9 and consider the two new integers formed i.e. 6 and 21. Add 6 and 21 and multiply it by 9

(6+21)*9 = 243 now add the remainder from the previous step. 243+8 = 251. Now write 1 in ten’s place of the answer and carry 25 to step 3. Answer: 11

Now multiply the two new integers formed. 6*21 = 126. Add the remainder 126+25 = 151

The Answer: 15111.

Let us now derive the General formula for Multiplication.

derivation:

Consider the example 348*85

Let 348 and 85 be x and y with 8 and 5 being u_1 and u_2 numerals in the units’ place respectively.
348*85 = 29580 = (340*80) + [(340*5) + (80*8)] + 8*5

29580 = 27200 + 2340 + 40

348*85 = [(348-8)(85-5)] + [(348-8)*5+(85-5)*8] + 8*5 corresponds to

\[ x * y = [(x - u_1)(y - u_2)] + [(x - u_1)u_2] + [(y - u_2)u_1] + u_1u_2. \]

This is the required general formula for Multiplication

Thus we have derived the general formula for the product of any two integers as:

\[ x * y = [(x - u_1)(y - u_2)] + [(x - u_1)u_2] + [(y - u_2)u_1] + u_1u_2. \]

More examples: 1445*1600

Step 1: 5*0 = 0. Write 0 in the unit’s place of the answer. No remainder Answer: 0

Step 2: Now consider 144 and 160. (144*0) + (160*5) +0 = 0+800+0 = 800. Now place 0 in the ten’s place of the answer and carry 80 to step 3.

Answer: 00

Step 3: 144*160 = 23040. 23040+80 = 23120

Answer: 2312000.

Multiplying any 2 decimal numbers:

Any two decimal numbers can be multiplied by first ignoring the decimal point, following the three step method as described above and then replacing the decimal point appropriately in the answer.

Example 1:

12.7 * 33.2

At first ignore the decimal point and multiply using the three step method.

127 * 332

Step 1: 7*2 = 14. Write 4 in the unit’s place of the answer. Carry 1 to step 2. Answer: 4

Step 2: consider 12 and 33. (12*2+33*7) = 255. Add 1 255+1 = 256.

Now write 6 in the ten’s place of the answer and carry 25 to step 3. Answer: 64

Step 3: 12*33 = 396. Add 25 from step 2. 396+25 = 421. Answer: 42164

Now count the number of digits on the right hand side after the decimal point of the two numbers considered.

12.7 has one digit after the decimal point and 33.2 has one digit as well. So the total is two.

Now place a decimal point after two digits from the right hand side of the answer.
Answer $42164 = 421.64$

And that is the required answer.

Example 2:

$4.65 \times 142.1$

Step 1: $1 \times 5 = 5$. Write 5 in the unit’s place of the answer. No remainder Answer: 5

Step 2: consider 46 and 142. $(46 \times 1 + 142 \times 5) = 756$. Now write 6 in the ten’s place of the answer and carry 75 to step 3. Answer: 65

Step 3: $46 \times 142 = 6532$. Add 75 from step 2. $6532 + 75 = 6607$. Answer: 660765

Total number of digits after the decimal point in the two numbers considered is three. Therefore place a decimal point after three digits from the right hand side in the answer.

Answer 660.765 is the required answer.

This formula & method hold good for multiplying any two numbers from 1 to infinity.

Conclusion: We have now derived the general formula for multiplication & also devised a handy three step method to multiply any two numbers starting from the SUTRA “Ekadhikyena Purvena”. Vedic Mathematics SUTRAs are versatile & very powerful spanning the entire universe of numbers from minus infinity to Infinity, yet are simple & easy to use. A thorough knowledge of Vedic Mathematics is imperative for one to become highly numerate & to develop excellent problem solving skills.