Abstract: This paper derives a quick method to divide any number by \(10^n - 1\) or monodigit number.

1. Basic Term

1) Monodigit Number

Number made up of single digit is called as monodigit number.

E.g.  
- i) 1111  
- ii) 22222  
- iii) 33  
- iv) 999999999  
- v) 8888888 etc.

Let decimal digit \(D\) occur \('n'\) times in monodigit number, then general algebraic representation of monodigit number can be derived as below

Monodigit Number = \(\underbrace{DDDD...DDD}_{n \text{ times}}\)

\[= D \times \underbrace{11111...111}_{n \text{ times}}\]

\[= \frac{D}{9} \times \underbrace{99999...999}_{n \text{ times}}\]

\[= \frac{D}{9} \times (10^n - 1)\]  (\(\because\) \(99999...999 = 10^n - 1\))

\[\therefore \text{ 'n digit' Monodigit Number} = \underbrace{DDDDD...,DDD}_{n \text{ times}} = \frac{D}{9} \times (10^n - 1)\]
2) Number’s Polynomial Representation

Let \( P = D_nD_{n-1}D_{n-2}D_{n-3} \ldots D_2D_1D_0 \cdot D_{-1}D_{-2}D_{-3} \ldots D_{-m+1}D_{-m} \) be number, which has \( n+1 \) digit before decimal and \( m \) digit after decimal, represented in number system with base \( b \). Then polynomial representation of number \( P \) is derived as below.

\[
P = D_n \cdot b^n + D_{n-1} \cdot b^{n-1} + D_{n-2} \cdot b^{n-2} + \ldots + D_2 \cdot b^2 + D_1 \cdot b^1 + D_0 \cdot b^0 + D_{-1} \cdot b^{-1} + D_{-2} \cdot b^{-2} + \ldots + D_{-m+1} \cdot b^{-m+1} + D_{-m} \cdot b^{-m}
\]

\[
P = \sum_{k=n}^m D_k \cdot b^k
\]

Above equation is called as polynomial representation of number \( P \) in base \( b \). Similarly if number \( P \) contains \( \infty \) number digits after decimal places and \( n+1 \) digit before decimal places, then its polynomial representation in base \( b \) becomes as below.

\[
P = \sum_{k=n}^{\infty} D_k \cdot b^k
\]

2. Method Formulation

Let \( y \) be dividend to which we want to divide by \( 10^n - 1 \) or \( 999999 \ldots 999 \), \( n \) times.

i.e.

\[
\frac{y}{(10^n - 1)} = \frac{y}{(x^n - 1)} \quad \text{(assuming } x = 10 \text{)}
\]

\[
= \frac{y}{x^n(1 - x^{-n})}
\]

\[
= \frac{w}{(1 - z^{-1})} \quad \text{(assuming } \frac{y}{x^n} = w \text{ and } x^n = z \text{ )}
\]

\[
= w \cdot (1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + \ldots)
\]

\[
= w \cdot \sum_{r=0}^{\infty} z^{-r}
\]
\[ w = \sum_{r=0}^{\infty} a_r z^r \] (substituting polynomial representation of \( w \) in base 'z' i.e. \( w = \sum_{r=0}^{\infty} a_r z^r \))

\[ w = (a_m)z^m + (a_{m-1} + a_m)z^{m-1} + (a_{m-2} + a_{m-1} + a_m)z^{m-2} + \cdots \] \hspace{1cm} \text{-------------(3)}

\[ w = \sum_{r=m}^{\infty} b_r z^r \] \hspace{1cm} \text{-------------(4)}

Comparing equation (3) & (4) we get

\[ b_m = a_m \]
\[ b_{m-1} = a_{m-1} + a_m = a_{m-1} + b_m \]
\[ b_{m-2} = a_{m-2} + a_{m-1} + a_m = a_{m-2} + b_{m-1} \]
\[ \vdots \]
\[ b_k = a_k + b_{k+1} \] \hspace{1cm} \text{-------------(5)}

Above equation gives required recurrence relation using which division by 999...999 or Monodigit number can be computed with magical speed.

3. Examples

1) 123475 ÷ 7777 =?

\[ 7777 = (7/9)(10^4 - 1) \]

Therefore, \( 123475 ÷ [(7/9) (10^4 - 1)] = 158753.5714285714 ÷ (10^4 - 1) \)

Comparing \( y ÷ (x^n - 1) \) where \( x = 10 \)

\[ \Rightarrow Y = 158753.5714285714...\quad n=4 \]
\[ Z = x^n = 10^4 \]
\[ \Rightarrow W = y/z = 15.87535714285714... \]
\[ = 15z^0 + 8753z^1 + 5714z^2 + 2857z^3 + 1428z^4 + \cdots \] \hspace{1cm} \text{-------------(I)}

Comparing equation (I) with \( w = \sum_{r=m}^{\infty} a_r z^r \) we get

\[ m=0, a_0 = 15, a_1 = 8753, a_2 = 5714, a_3 = 2857, a_4 = 1428 \text{ etc.} \]
\[
\begin{align*}
\Rightarrow \quad b_0 &= a_0 = 15 \quad \text{(using equation (5))} \\
b_1 &= a_1 + b_0 = 8753 + 15 = 8768 \\
b_2 &= a_2 + b_1 = 5714 + 8768 = 14482 \\
b_3 &= a_3 + b_2 = 2857 + 14482 = 17339 \\
b_4 &= a_4 + b_3 = 1428 + 17339 = 18767 \\
\vdots \\
\end{align*}
\]
So on.

Then, according to equation (4) resulting division is given as below:

\[y = \frac{x^n - 1}{(x - a)} = \left( \sum_{r=m}^{\infty} b_r \cdot z^r \right)\]

\[= b_0 \cdot z^0 + b_1 \cdot z^1 + b_2 \cdot z^2 + b_3 \cdot z^3 + \ldots\]

\[= 15 \cdot z^0 + 8768 \cdot z^1 + 14482 \cdot z^2 + 17339 \cdot z^3 + 18767 \cdot z^4 + \ldots\]

\[= 15.8769 \text{ 4483 7340 8767 (Since } Z=10^4)\]

\[\therefore 123475 \div 7777 = 15.8769 \text{ 4483 7340 8767}\]

b) Shortcut Method

\[7777 = 7 \cdot \left( \frac{10^4 - 1}{9} \right)\]

\[\therefore \frac{123475}{7777} = \frac{12345 \cdot 9}{7} \cdot \frac{1}{10^4 \cdot (1 - 10^{-4})}\]

\[= 15.8753 \text{ 5714 2857 1428...}\]

\[+ \]

\[= 15. \text{ 8769 4483 7340 8767...}\]

OR
\[ \frac{123475}{7777} = 15.8769448373408768 \ldots \]

2) \( \frac{2367428}{142857} = ? \)

\[ \text{We know that } \frac{1}{7} = 0.142857 = \frac{142857}{999999} \]

\[ \therefore 142857 = \frac{999999}{7} \]

\[ \Rightarrow \frac{2367428}{142857} = \frac{2367428\times7}{999999} = \frac{16571996}{999999} = \frac{16571996}{10^6 \times (1-10^{-6})} = \frac{16.571996}{1-10^{-6}} \]

\[ \therefore \frac{2367428}{142857} = \frac{16.571996}{1-10^{-6}} \]

\[ \frac{16.571996}{1-10^{-6}} \] can be computed by using above shortcut method as below.

\[ \begin{array}{c}
16. 571996 00000 00000 00000
+ 000016 572012 572012
\hline 16. 572012 572012 572012 572012
\end{array} \]

\[ \therefore 2367428 \div 142857 = 16.572012 \]