From Nathan Annenberg:

A Way to Better Focus the VM Base Division Algorithm

After you master flag division, there is yet another level of complexity involving huge divisors, near higher powers of 10 bases, like 9845 of 12332. VM has provided us a way to sail through these – however, depending on the conditions of the problem, you get either elation or disaster, where, in the latter case, you don’t sail, you sink fast.

Ex. of success

\[
\begin{array}{c|cc}
986 & 84 & 167 \\
014 & 1 & 12 \\
\hline
85 & 357 \\
\end{array}
\]

So our 986 has an offset from its base of 1000 of 014. We must preserve this zero to force it to travel as three places because the base is \(10^3\). This also explains why our vertical demarcation line is 3 places leftward from the right edge of the dividend. We bring down the 8 and X by the 014 offset for our first addend of 112. Then \((4 + 1)\) \(X\) the offset = 070 which shifts one place over to the right. Since we are now entirely within the remainder block, we are already done. 85 R357

Ex. of chaos

\[
\begin{array}{c|cc}
923 & 88 & 665 \\
077 & 6 & 16 \\
\hline
814 & 1903 \\
94 & 1903 \\
96 & R57 \\
\end{array}
\]

I am going to space the dividend digits apart here – believe me it \textit{will} be necessary.
Bring down the 8 and X the 077 offset = 616
Add that and you get a 2-digit 14 next to the quotient 8. You
cannot carry yet till you have done 14 X 077 = 1078 in remainder
block. Only now can you carry to get 94 and sum of all those
remainders = 1903.

So far 94 R 1903: Now you must extract all the divisor
923s you can from this huge total. You get 2 extra 923s for a
quotient of 96 and a final net remainder of 57.

We solved it but it was cumbersome. Try this with
flag division and it should be a lot simpler.

There were many things that complicated this
problem:
- A large offset, though not terrible if other conditions were
  better.
- A dividend that should have been further back relatively,
  from its base than the divisor
- A key digit in the remainder part of the dividend that did
  not lend well to avoiding these giant remainders.

How to prevent chaos like this? I worked out a system of 4 rules
that perhaps 95% of the time results in a smooth, seamless
solution process.

How to Discriminate Properly Which Problems
Are Amenable to Rapid Solution
For the “Shifting Addends” Method.

There seem to be 4 rules that if taken together, pretty much
guarantee success in a complication-free quotient. Even 3 out
of these 4 often work.

Rule 1 – The divisor is close to its base – i.e. very small to
moderately small offsets. There should be at least one zero in
the offset placeholders, which will slow down the accumulation
of addends.
Rule 2: The dividend is relatively further away from its base than the divisor is to its own base – the more, the better. This allows more addition if under base, and more subtraction if over base.

Rule 3: The first digit in the dividend starting the remainder block is low if divisor is under its base, and high if divisor is over its base. (We will call this the “cutoff digit”) This rule is particularly crucial. It prevents excess addition or subtraction in the column transitioning to the quotient block.

Rule 4: The number of digits in the dividend is no more than 3 more than those of the divisor’s power of 10. This prevents too many rows of shifting addends.

Ex. 1: both under base

\[
\begin{array}{c}
  997/89223 \\
  997 \\
  003 \\
  \underline{027} \\
  89490 \\
\end{array}
\]

Comments: small offset \( \rightarrow \) 3, which includes two zeroes, dividend farther from base than divisor, cutoff digit = 2, low to allow adding without carry overflow, divisor only two digits less than dividend so just two partial addends.
Ex. 2 Both over base

\[
\begin{array}{cccccccc}
1 & 0 & 1 & 3 & 1 & 3 & 4 & 9 & 5 & 6 \\
(0 & 1 & 3) & 0(1) & (3) & 0 & (3) & (9) & 0 & (3) & (9) \\
1 & 3 & 3 & 3 & (7) & (3) \\
1 & 3 & 3 & 2 & 2 & 7 & 133 & R & 227 \\
\end{array}
\]

Comments: moderately low offset (013): i.e. one zero, dividend well above divisor, cutoff digit = 9 to allow several rows of negative addends, and dividend is only 3 more digits than divisor for 3 rows of negative addends. While there is negative overflow toward the right, the KEY column, that of the cutoff digit, survives intact as a positive value.

Ex. 3 Mixed number types: divisor under base, dividend over base

\[
\begin{array}{cccccccc}
9 & 8 & 5 & 3 & 2 & 3 & 5 & 6 \\
0 & 1 & 5 & 0 & 4 & 5 & 0 & 3 & 0 \\
3 & 2 & 8 & 3 & 6 & 32 & R & 386 \\
\end{array}
\]

Comments: Because of just one zero in the offset, I was able to risk a cutoff digit that was almost mid-level. However, the two-digit difference between divisor and dividend definitely helped, leaving only two layers of shifting addends.

Ex. 4 Mixed number types: divisor over base, dividend under base
Comments: We had a little luck helping us here. The high cutoff digit 8 would not have survived 3 negative shifting addends. Fortunately, the first addend was limited entirely to the quotient side. Also, the bar 5 within the quotient produced a positive shifting addend which cancelled much of the negative effect. I was taking a chance with a three-digit difference between the 5-digit dividend and the 2 digit places from divisor’s power of 10^2, which resulted in 3 shifting addends, a risky thing to do when you have a 9 in the dividend (as well as the divisor here) which can magnify things very quickly.

Additional Observation: Occasionally, even with all this, you might still get carry overload in the remainder block. But it’s easy to translate to the proper value because of the small divisor offsets. Ex. Divisor is 992, quotient is 6 and remainder is 1106. So extract out a 992 for a quotient of 7, and do backwards addition towards the 1106 for the true remainder i.e. 992 + offset 8 = 1000, then add the 106 for remainder of 114.