

A NEW SQUARE- ROOT ALGORITHM FOR LARGE NUMBERS

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Abstract: The potential of the Sutra “Ekadhikena Purvena” of Vedic Mathematics has been explored to obtain the square root and square of any positive real number. The algorithm developed has been demonstrated on different types of numbers. A reader who understands simple arithmetic operations would follow the newly developed methodology.

Keywords: Vedic mathematics, square-root, square, real numbers.

1. Introduction

Mathematical science has witnessed much advancement which aided the computerization, especially in the number theory. The difficulty in handling numbers in the decimal system has led to the development of binary system. The numerical techniques and the number theory render procedures to minimize the computational time. The real time computational algorithms are received with appreciations by the stalwarts in computer science. If the development of fast computers in terms of hardware is on one hand the maximizing the computational efficiency with reduction in elapsed time is the role of software giants on the other hand. The common domain

for the two traditions of computer science and technologies is the mathematical science. The computer science and the mathematical science go hand in hand in the current on going scenario. More thrust is given to the development of parallel computers and parallel algorithms to effect the reduction in size of computers and to handle large size problems and data with the available hardware.

The contributions of the Indian ancient mathematicians in the world history of mathematical science are not well recognized. The depth of the mathematical formulation in the form of “Sutras” presented by the Indian saints, hundreds of year back has not been gauged and explored to the fullest extent. The buried treasure in the Indian mythological statements is so wide and great that they contain absolute perfection that can propel the current science to the great heights. It is well known that Everest is the only one and not many have scaled it, which is surrounded by many other mountains with in easy reach. An ultimate without approximation and with real time touch of simple mathematics has always a say that is attractive like a fountain. It is not totally ruled out that efforts are not made to expose the authenticated philosophy of Indian science and mathematics. But the examples are few even though the profiles are many. There are countable personalities who got recognition in the global scenario. The downward slide in quality research efforts of the Indian scientists and technologists, is by the lack in financial aid and support for the potential thinkers.

The contributions of the saint and mathematician to the number theory, by Sri Bharati Krasan Thirtha Swamiji, in the form of Vedic Sutras are considerable. He had explored the mathematical potentials in the Vedic primers and showed that the mathematical operations can be carried out mentally to produce fast answers using the Sutras; some of the Vedic sutras cited in¹ demonstrate their power in multiplication, division and squaring and finding square roots etc. One of the sutras discovered by

Swamiji is churned in this article with due respect to the great saint, to bring out a formulation that can give the square root and square of any number given. As suggested by Swamiji in¹ the sutra “Ekadhikena Purvena” can give the square of a number ending with 5. But when the particular sutra meaning ‘one more than the previous’ is gauged, it went to such a depth that squaring of any digits number and square rooting any given number is possible in a fascinating way. The objective of this article is to prove an algorithm to generate the square root of any given number using the Vedic sutra “Ekadhikena Purvena” and to show that the reverse procedure can give the square of any number. The computational algorithm exhibit great amount of parallelism in its workout, and utilizes simple arithmetic operations like addition, subtraction, multiplication and division. Even though the algorithm is not computerized, it can be authoritatively stated that the formulation can give fastest result. As far as the accuracy is concerned, the results obtained are absolute and are limited by hardware constraints.

In the process a ‘Y’ operator (Yojayet operator) has been defined and utilized, in the algebraic formulation of the sutra, “Ekadhikena Purvena”. The inverse Y (Y^{-1}) defined eased the process of finding the square root. Even though the effort looks like an old wine in a new bottle; it is the clear exploration of the left out streak line in the vast computational gush that can be accepted in the right spirit of resolution.

The actual application of the sutra considered, as explained by meditating Swamiji is presented to highlight the explored potential of the sutra. Further the attempt is made to define a new Y operator and its inverse. The utility of the operators is discussed subsequently. Finally the formulation of the algorithm to find out the square root of any number is proved and demonstrated with various examples. Nevertheless the inverse algorithm can provide the square of any number which is

also exemplified to highlight the supremacy of the formulation. The development presented in this article is new and different to the knowledge of the author. As mathematics provides the tool for the laws of science, the relevance of the topic may be well received.

2. Formulation

Analysis of the sutra- “Ekadhikena Purvena”

The literal meaning of the stated sutra discovered from Veda by spiritual Swamiji means “one more than the previous”. He successfully applies this sutra for finding the square of a number ending with 5. Suppose if we are to find the square of 85 the previous number is 8. One more than the previous is 9. Multiply 8 and 9 and unite it with square of 5 to get square of 85 equal to 7225. To generalize the statement algebraically, if ‘m5’ is the number to be squared then

$$(m5)^2 = m(m+1) \times 100 + 25 \dots \dots \dots (1)$$

Definition of Y (Yojayet) operator

By definition Y operator is the combination of two operations embedded to simply the handling of the sutra. Mathematically it is stated as

$$Y_n = (\times 10^{n-1})_+$$

Hence the statement (1) considering ‘m’ to be a previous number, can be written as

$$(mY_2 5)^2 = m(m+1)Y_3 25 \dots \dots \dots (2)$$

For example,

$$3Y_n 8 = 38 \quad \text{if } n = 2$$

$$= 308 \quad \text{if } n = 3 \quad \text{and so on.}$$

Hence the sutra, Ekadhikena is a concise primer that unconditionally provides a square of any number ending with 5. The statement (2) is the sutra for better understanding. If we have to square 105, then

$$(10Y_2 5)^2 = 10(10+1)Y_3 25$$

$$= 110 \times 100 + 25 = 11025 .$$

Definition of inverse Y operator

Reverse process of the sutra is envisaged by the opposite of the operations in the Y operator. Hence inverse Y operator can be presented as

$$Y_n^{-1} = -25 \times 10^{-(n-1)}$$

Therefore

$$Y_3^{-1} 2025 = \frac{(2025 - 25)}{10^{(3-1)}} = 20$$

Square root of a number

The west component, 'b' of a number is obtained using the Y^{-1} operator and the terminal component (i.e., T=25) as,

West component of $N = b = Y_3^{-1}N = \frac{N-25}{10^{3-1}} = \frac{N-25}{100}$

Its clear from the sutra Ekadhikena Purvena that

$$m(m+1) = b \dots\dots\dots (3)$$

Hence the discreminant component is written as

$$D = \pm\sqrt{4b+1} \dots\dots\dots (4)$$

The root component of equation (3) is given by

$$m = \frac{D-1}{2} \dots\dots\dots (5)$$

Now the square root is given by

$$\sqrt{N} = mY_2|\sqrt{T}| \dots\dots\dots (6)$$

Comprehensively the formula is

$$\sqrt{N} = \left(\frac{\pm \left[\sqrt{4Y_3^{-1}N+1} \right] - 1}{2} \right) \times 10 + 5$$

Examples of demonstration

Consider a number which is a complete square ending with 25, say 11025.

Terminal component, $b=Y_3^{-1}11025$

$$=110$$

The discreminant, $D = \sqrt{4b+1} = \sqrt{441} = 21$

The root component, $m = \frac{m-1}{2} = \frac{21-1}{2} = 10$

The square root of 11025 = $mY_2 \left| \sqrt{T} \right|$
 $= 10Y_2 5$
 $= 10 \times 10^{2-1} + 5 = 105.$

Now let us consider a complete square not ending with 25, say 8836

Terminal component of 8836 = $Y_3^{-1} 8836$

$$b = 88.11$$

The discriminant, $D = \sqrt{4 \times 88.11 + 1}$
 $= \sqrt{353.44} = 18.8$

The root component = $m = \frac{D-1}{2} = 8.9$

The root = $mY_2 \left| \sqrt{T} \right| = 8.9 \times 10 + 5 = 94.$

An arbitrary number which is not a complete square may be considered for square rooting say 1239

Terminal component, $T=25$

West component of 1239 = $Y_3^{-1} 1239$

$$b = 12.14$$

The discriminant, $D = \sqrt{4b + 1} = \sqrt{49.56} = 7.039886363$

The root component, $m = \frac{D-1}{2} = 3.019943181$

The root is $mY_2 \left| \sqrt{T} \right| = m \times 10 + 5 = 35.19943181$

It may be verified that $(35.19943181)^2 = 1239$.

As a demonstration on a large number consider the square root of 100100025

The terminal component is, $T=25$

The west component is $Y_3^{-1} 100100025 = 1001000$

The discriminant, $D = \sqrt{4004001}$

At this instance the square rooting algorithm may be applied to D itself in a looping arrangement of operation.

Hence $D= 2001$.

The root component, $m = \frac{2001-1}{2} = 1000$

The square root is $mY_2 5 = 1000 \times 10 + 5 = 10005$

To consider the square of a fractional number, say 632.89

Terminal component, $T= 25$

West component, $b = 6.0789$

The discriminant, $D = \sqrt{4 \times 6.0789 + 1} = 5.03146102$

The root component, $m = \frac{m-1}{2} = 2.01573051$

The square root is $mY_2 5 = 25.1573051$

It may be verified that square of 25.1573051 is 632.89.

Square of any number

By operating the square root algorithm in the reverse direction yields the square.

Subtract the terminal component '5' from the given number and multiply by 1/10.

$$m = \frac{N - 5}{10}$$

Find the discriminant

$$D = 2m + 1$$

Find the west component

$$b = \left(\frac{D^2 - 1}{4} \right)$$

Find the square of the number using

$$bY_3 25 = b \times 10^2 + 25$$

Example: A single decimal number, say 47.8 may be considered for squaring

(N= 47.8)

- $m = \frac{47.8 - 5}{10} = 4.28$
- $D = 9.56 (= 2 \times 4.28 + 1)$
- $b = \frac{(9.56)^2 - 1}{4} = 22.5984$
- $bY_3 25 = 2284.84$

3. Conclusion

With a salutation to the spiritual saint Sri Bharati Krasna Thirtha Swamiji who with his insight related Vedic primers with mental mathematic calculations, one of the starter sutras is taken as the foundational basis. Fascinated by the marvel of the sutra, Ekadhikena Purvena, a furthering effort of digging the mine of immense treasure, has led to the algebraic formulation and development of algorithm that calculates the square root and square of any given number, positive integer or real

of any size. Limitation on accuracy is the hardware. The generalized formulation contains the definition of a new operator Y (Yojayet) and its inverse Y^{-1} to make the carryout different and handy. It is anticipated that the newly defined operator can open a way for design and development of a different device that may lead to areal time embedded system. It is envisaged that super-ultra-hyper speed systems are not far of from reach. This is the initial flick of current science showing the direction for the future science on a mathematical basis.

The directed effort is illustrated through different and diversified examples to remove the doubt of non-conformity and exceptionality, and to demonstrate the generalized supremacy. This is submitted with great pleasure as a presentation of an Indian asset to appear in an International scenario. Spring of Indian philosophy never dries.

Reference:

1. Jagadguru Swami Sri Bharati Krsna Thirthaji Maharaja, "Vedic Mathematics", Motilal Banarasidas Publishers Pvt Ltd., Delhi, (1994).

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